Text reference pages 180 and 268.

The `rank` Command

Purpose
To study uses of the `rank` command.

MATLAB Functions
`rank, r == s`

On page 178 of the text, rank A is defined to be the dimension of Col A. MATLAB has a built-in function `rank(A)` for computing rank. This function greatly simplifies certain computations; for example, to check if the columns of `A` are linearly independent, you need only check if `rank(A)` is the same as the number of columns. Similarly, the easiest way to check if $\mathbf{v}$ is a linear combination of the columns of A is to check if the rank of A is the same as the rank of $[A, \mathbf{v}]$. The MATLAB function `r == s` tests if the two floating point numbers are identical, returning a `1` if they are identical and returning a `0` if they are not. Thus, to test if $\mathbf{v}$ is in the column space of A, check if the following returns `1`.

```
rank(A) == rank([A,v])
```

To check if the columns of B form a basis for Col A we need
(1) To check that the columns of B are in Col A:

```
rank([A,B]) == rank(A)
```

(2) To check that the columns of B are linearly independent

```
rank(B) == n
```

where n is the number of columns of B.
(3) To check that the number of columns matches the dimension of Col A

```
rank(A) == n
```

To check Col A = Col B, simply check both

```
rank([A,B]) == rank(A)
```

and

```
rank([B,A]) == rank(A)
```

There is no (nor can there be any) MATLAB function `dim`, which would find the dimension (of what?). The most obvious difficulty would be that we would not know which subspace Col A or Nul A, to mention two of the several possibilities, that `dim(A)` might refer to. The `rank` command can help us sort out the dimensions of these subspaces. The dimension of the column space of `A` is given by

```
rank(A)
```

The dimension of the null space of `A` is given by

```
size(A,2) - rank(A)
```

where `size(A,2)` is MATLAB's command for finding the number of columns of `A`. What is `size(A,1)`?

MATLAB has particular conventions about the rank of certain matrics. The matrix

```
A = ones(5) + 2 ^ (-60)*eye(5)
```

is theoretically invertible, and so has rank 5, but look at

```
rank(A)
```

MATLAB's result here is perfectly justified. From a computational point of view, the matrix is not substantially different from `A = ones(5)` which has rank 1.

MATLAB Exercises

1. Let `A = magic(8)` and compute the following:
   a. dim Col A
   b. dim Nul A
   c. dim Col A^T
   d. dim Nul A^T

2. Determine if the columns of B form a basis for Col A where `A = rand(5,3)` and `B = A*rand(3,3)`.

3. For `A = rand(3,5)` and `B = null(A)*rand(2,2)`, determine if the columns of B form a basis for Nul A where

4. Determine if Col A = Col B where `A = rand(6,2)` and `B = A*rand(2,3)`.

5. Determine if Col A = Nul B where `B = rand(2,6)` and `A = null(B)*rand(4,3)`.

6. Use MATLAB's `rank` command to determine which of the following matrices are invertible:
   a. `magic(4)`
   b. `magic(5)`
   c. `magic(6)`
   d. `magic(7)`

7. Compute the rank of the following matrix `A = ones(5) + 2 ^(-k*10)*eye(5)`, for `k = 1:8`.